ON BEDNOV'S ARTICLE "A NEW METHOD FOR SOLUTION OF THE INTEGRAL EQUATIONS FOR RADIANT HEAT EXCHANGE"

S. P. Rusin

The abstract of [1] stated that that study "presented a method for calculating radiant heat exchange and a new approach to solution of the corresponding equations." From such a study the reader has the right to expect a new departure in methods for solving the integral equations of radiant heat exchange.

The entire study dealt with an example consisting of a system of two diffusely radiating and reflecting grey surfaces in the form of two bands of infinite length located one above the other and having the same width. The fundamental model problem was solved in the following formulation: the surfaces F_1 and F_2 are isothermal temperatures and radiating (ε_i) and reflective (R_i) capabilities of the surfaces are specified. It is required to find the distribution $E_{ef}(M_i)$ ($M_i \in F_i$, $i = 1, 2$) on each of the surfaces (the notation being the same as that of [2]).

With no explanation, apparently following [3] (p. 214, Eq. 5.69)), the author writes expressions for $E_{ef}(M_i)$:

$$E_{ef}(M_i) = \sum_{j=1}^{2} f_{ij}(M_i) E_{c,j}, \quad i = 1, 2, \tag{1}$$

where $E_{c,j}$ is the natural radiation structure density F_j. Then, transforming a system of integral equations of the form

$$E_{ef}(M_i) = E_{c,i} \int_{F_j} K(M_i, N_j) E_{ef}(N_j) dF_{N_j}, \quad i = 1,\ j = 2 \text{ and } i = 2,\ j = 1, \tag{2}$$

with the aid of Eq. (1) and utilizing the arbitrariness of $E_{c,i}$ the author obtained a system of four integral equations in the unknown functions f_{ij} ($i, j = 1, 2$) which decomposes into two independent systems soluble by a single algorithm. Then, by making use of the unique features of the geometry of the given system of surfaces the author proposed to solve not two, but only one system of equations.

This approach is not a new one, as I shall show. The physical sense of the functions f_{ij} is optical-geometric and they can be written in terms of angular resolution coefficients Φ_{ij} [4], which have been used for solution of similar and more complex problems for more than a decade. Then Eq. (1) can be written as:

$$E_{ef}(M_i) = E_{c,i} + R_i \sum_{j=1}^{2} \Phi_{ij}(M_i) E_{c,j}, \quad i = 1, 2, \tag{3}$$

where the angular resolution coefficients Φ_{ij} characterize the fraction of natural radiation of an elementary area at the point M_i which falls on the surface F_j directly as a result of multiple reflections within the system. Thus, as in [4] the problem reduces to determination of Φ_{ij} from a system of four integral equations of the type

$$\Phi_{ik}(M_i) = \varphi(M_i, F_k) + \sum_{j=1}^{2} R_j \int_{F_j} (M_i, P_j) \Phi_{jk}(P_j) dF_{P_j}, \quad i, k = 1, 2, \tag{4}$$

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 55, No. 6, pp. 1039-1040, December, 1988.

0022-0841/88/5506-1451$12.50

where φ is the local angular coefficient.

It should be noted that system (4) is more general than Eqs. (6) and (9) of [1], since it characterizes heat exchange by radiation in a system of several surfaces. Moreover, the angular resolution coefficients have the property of reciprocity and closure, while the resolvents have the property of symmetry, which is important in reducing the volume of calculations. It is simple to show that Eqs. (6) and (9) of [1] are a special case of system (4) for $K(M_i, N_i) = 0$ and $K(M_i, N_i) = K(M_i, N_j) = K(M_j, N_i)$.

It is well known that if the unknown integrand is constant then an integral equation of the form of Eq. (2) is identically algebraic and can be solved exactly. In essence no new approach is proposed in [1]. The problem posed by the author can be solved more effectively using the widely employed apparatus of angular resolution coefficients developed by Surinov and his school [4, 5]. For isothermal symmetric plane radiating systems of two bodies Bednov in fact obtained for the local angular resolution coefficients integral equations which are a special case of the more general integral equations presented in [4]. In addition it should be noted that [1] once again demonstrates the fact that use of unique geometric and thermal features of particular radiating systems allows the number of integral equations required for determination of the angular resolution coefficients to be reduced, and this is the positive side of the study.

LITERATURE CITED

1. S. M. Bednov, Inzh.-Fiz. Zh., 51, No. 6, 1031-1040 (1986).
2. A. S. Nevskii, Radiant Heat Exchange in Furnaces and Burners [in Russian], Moscow (1971).
3. M. N. Otsisik, Complex Heat Exchange [in Russian], Moscow (1976).
4. Yu. A. Surinov, Dokl. Akad. Nauk SSSR, 83, No. 2, 223-226 (1952).
5. Yu. A. Surinov, Tr. Univ. Druzh. Narod. P. Lumumby, 12, No. 2, 82-130 (1965).

REPLY TO S. P. RUSIN

S. M. Bednov

The remarks made in the above letter concerning my study indicate the author's deep understanding of the questions touched upon by the original article. However, in my opinion, those remarks relate more to the form of the exposition than to the actual content of the article. Some of the relationships presented in the original article were in fact presented without explanation in light of the simplicity with which they can be derived. System (4) of the current letter is in fact more general than Eqs. (6) and (9) of the original article.

However the goal of the study in question was not at all to construct the most general system of equations describing heat exchange by radiation in a system of bodies — those expressions were derived long ago. The new feature of the study was the approach to solving those well known equations, consisting in calling attention to the asymptotic behavior of the integrands, which allows, at least for the system of bodies considered, creation of approximate (and in limiting cases, exact) analytical solutions coinciding with satisfactory accuracy with both available experimental data and numerical solutions.

As for the more effective solution of such problems with "the aid of the widely employed apparatus of angular resolution coefficients developed by Surinov and his school," I could find no such apparatus in the references [4, 5] cited in the letter. In the general case, as was noted in [1] (p. 66), "... determination of the resolent of the form Γ(M, N)* from the corresponding integral equation presents a more complex problem than direct numerical solution of the original integral equation." In simpler cases these coefficients can be found numerically with the aid of either infinite series or initial analog systems of integral equations, or experimentally with the aid of light or electrical modeling.

LITERATURE CITED

1. S. P. Rusin and V. E. Peletskii, Thermal Radiation of Cavities [in Russian], Moscow (1987).

---

* $\Phi(M_i, F_j) = \int\limits_{F_j} \Gamma(M_i, N_j)\, dF_N$, $\Phi_{ij} = \frac{1}{F_j} \int\limits_{F_i} \int\limits_{F_j} \Gamma(M_i N_j)\, dF_{N_j} dF_{M_i}$ in the same notation used in the letter and [1].

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 55, No. 6, pp. 1040-1041, December, 1988.

